Child Labor, Fertility, and Economic Growth

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Abstract

This paper explores the relations between child labor, fertility and growth. The existence of child labor can explain the high level of fertility and low output per capita due to low levels of human capital, which characterizes economies in a development trap. Technological progress that occurs endogenously pulls the economy out of the trap. Along the dynamics to a new steady state, parents find it optimal to substitute child quality for quantity. Hence, child labor and fertility decline while schooling increases and consequently consumption and output increase. At the new steady state, child labor is abolished and fertility is low, whereas consumption and output grow forever. Hence, child labor contributes a new explanation of demographic transition via the mechanism of a change in the relative incomes of children and parents.

The Maurice Falk Institute for Economic Research in Israel
Jerusalem, July 1999  •  Discussion Paper No. 99.02
CHILD LABOR, FERTILITY, AND ECONOMIC GROWTH*

1. Introduction

Child labor is a mass phenomenon in today’s world. According to the ILO Bureau of Statistics, 250 million children aged 5–14 were economically active in 1995, almost a quarter of the children in this age group world-wide. The phenomenon is most widespread in the poorest continent, Africa, but was not always the sole province of less developed countries: child labor was once common in Europe and in the United States, too. In 1851 England and Wales, 36.6 percent of all boys aged 10–14 and 19.9 percent of girls in the same age group worked. The historical evidence suggests that child labor has been part of the labor scene since time immemorial. The industrial revolution only highlighted the plight of working children because this was probably the first time that child labor took place outside the household and became wage labor.

An abundance of literature on modern-day child labor is available, both in the field of economics and in other disciplines. Goldin (1979) explored the phenomenon in late 19th-century Philadelphia, finding that a father’s full-time wage was negatively cor-

* We are grateful to Oded Galor, Joseph Zeira, and seminar participants at the Hebrew University Department of Economics for their helpful comments.
3 Cunningham (1990) cites many historians who provide evidence that child labor was common in England before the industrial revolution.
4 Most of the literature is in Social Work, refers to the exploitation and abuse of children, and therefore views child labor as a problem to be solved. See Rodgers and Standing (1981) and Bequele and Boyen (1988). Weiner (1991) presents a Political Science approach to child labor in India.
related with his child’s probability of working. Levy (1985) studied the relationship between child labor and fertility in rural Egypt, focusing on the economic contribution of children and its effect on desired family size and fertility among farmers. Levy’s work supports Goldin’s result: he finds that an income effect leads to an increase in the demand for children’s leisure and schooling. Levy’s work also identifies causality between child labor and fertility, supporting the hypothesis that the more significant a child’s contribution to family income, the larger the desired family size. Cain and Mozumder (1981, p. 280), who studied labor market structure and reproductive behavior in rural south Asia, concluded that “… the costs of children and their contributions to the household economy have direct relevance for fertility decisions.”

Theoretical models on child labor are, however, scarce. One such recent study (Basu and Van, 1998) presents a model that depends on two assumptions: (a) that child labor can be substituted by adult labor, and (b) that a child’s leisure is a luxury good. The result of the model is multiple equilibria in the labor market — “one equilibrium where children work and another where the adult wage is high and children do not work” (Basu and Van, 1998, p. 412). This model suffers from two shortcomings: the absence of dynamics (hence, it has no implication for economic growth), and lack of fertility choice. Lloyd (1994) suggests that larger household size reduce parents’ investment in schooling. Grootaert and Kanbur (1995) concluded from Lloyd’s work that the larger the household, the higher the probability that a child will work. Hence, based on Levy (1985), Cain and Mozumder (1981), and Grootaert and Kanbur (1995), it seems natural to explore the joint decision of fertility and child labor.

Theoretical models of fertility and growth are far more popular. Most of the literature tends to explore the negative relation between income and population growth that has prevailed in developed countries since the mid-19th century. Becker, Murphy and Tamura (1990) assumed that a high aggregate level of human capital raises the return to human capital. They also assumed that the return to children is decreasing in the number
of children. Since child rearing is highly time-intensive, whenever the aggregate level of human capital is high, fertility is low, and each child is provided with a high level of human capital. Galor and Weil (1996) present a different link between fertility and economic growth: They assumed that physical capital is more complementary to women’s labor input than to men’s, concluding that an increase in capital per worker raises the relative income of women, which increases the cost of children more than the increase in family income, resulting in a fertility decline. The decline in fertility, in turn, raises the level of capital per worker even further, and so on. This loop leads to a decline in fertility accompanied by a higher level of output.

The present paper addresses child labor and fertility within a unified theoretical model in order to examine the joint dynamics of child labor, fertility choice and accumulation of human capital and its impact on economic growth. We construct a framework in which child labor, schooling, and fertility/labor supply choice are jointly determined by households, which is consistent with the empirical evidence.

We agree with Basu and Van (1998), and argue that children participate in the labor force not because parents are necessarily cruel or selfish, but because the survival of the household depends on these children’s work. We show that child labor tends to decrease as the household’s crucial need for the income children generate diminishes.

The synthesis between fertility and child labor does not alter the results regarding fertility and growth, but rather supports it. Whereas it is optimal for households to reduce the number of children as the economy develops, the introduction of child labor offers a novel mechanism that produces a negative effect between fertility and growth. We will study how changes in parents’ relative income (vis-à-vis children’s income)

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5 It does not matter whether children work for wages to support the household or help in domestic work, releasing other household members to supply labor in the market.
affects fertility and child labor/schooling choices, and how these choices, in turn, affect economic growth via the supply of production factors.

Our model is based on empirical evidence mentioned above. Technological progress increases the relative income of the parent, which in turn has two effects: fertility declines and children’s schooling increases. These effects increase the relative income of the parent further and thus, along the dynamic path to steady state, families become smaller and better educated.

The reason for these effects is as follows: we define the individual’s preferences over two goods: (a) consumption and (b) “total quality” of children, defined as the children’s combined potential income in the next period (which is viewed as a reflection of parents’ concern for their offspring). Using log-linear preferences, income is divided proportionally between consumption and “children’s quality” according to the coefficients of the utility function. Since child-rearing is costly in time, the price of a child is indexed to the parent’s income. When the parent’s relative income increases, the price of a child increases as well, and fertility declines. On the other hand, since the cost of schooling is the opportunity cost in the labor market, the relative price of schooling (to childrearing) declines as the parent’s relative income increases. The above mechanism thus results in substitution between the quantity and quality (i.e., level of human capital) of children.

As in Galor and Weil (1996), the model also produces an endogenous demographic transition. As long as the technological level is low, fertility is high and constant, because the parent’s relative income is low and constant. However, once technology reaches a certain level, the economy experiences a dramatic change. Parents’ relative income begins to increase and the process described above “kicks in”: fertility declines

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6 From Section 2 it follows that the parent’s relative income can increase without technological progress, but when such progress is introduced, this relative income must increase.
and children’s schooling increases. These effects increase the relative income of the parent further and thus, along the path to steady state, families become smaller and better educated.

The paper is organized as follows: Section 2 presents the basic model of child labor and fertility with constant technology and derives the dynamic system implied by the model. In Section 3 we introduce technological progress (exogenous and endogenous), and analyze the resultant dynamic system. Section 4 concludes.

2. The Basic Structure of the Model

Consider a small, open, overlapping-generations economy that operates in a perfectly competitive world and faces a given world rate of interest. Time is infinite and discrete. In every period, the economy produces a single good that can be used for either consumption or investment. Three factors of production exist in the economy: physical capital, raw units of labor, and efficiency units of labor.

2.1. Production

In each period, there are two potential sectors. Production can take place in either one of them or in both. It is important to emphasize that the existence of one sector is independent of the existence of the other, and, as will become clearer later, the existence of each sector is determined by individuals’ optimal choices. In both sectors technology has constant returns to scale, but employs different factors: one technology employs only raw labor, while the other employs physical capital and efficiency units of labor. We refer to the former as “traditional” and to the later as “modern.”

7 The existence of these two sectors can represent the process of urbanization. Thus, the traditional sector can represent rural production and the modern sector can represent industrial
The production function of the traditional sector is:

\[ Y_{t, t} = w^c L_t ; \]  

the modern production function satisfies all the neoclassical assumptions and is given by:

\[ Y_{2, t} = F(K_t, H_t) , \]  

where \( L_t, H_t, \) and \( K_t \) are the quantities of raw labor, efficiency labor and physical capital, respectively, employed at time \( t , \) and \( w^c > 0 \) is the marginal productivity in the traditional sector. Given the production technology, the competitiveness of markets and the world interest rate, \( \bar{\tau} , \) firms’ inverse demand function for capital is:

\[ \bar{\tau} = f(k_t) , \]  

where \( k_t = (K_t / H_t) , \) and therefore,

\[ k_t = f^{-1}(\bar{\tau}) \equiv \bar{k} . \]

The return to one unit of raw labor is \( w^c \) and the return to one unit of efficiency labor, \( w_t , \) is:

\[ w_t = f(\bar{k}) - f(\bar{k}) \bar{k} \equiv \bar{w} . \]

2.2. Individuals

In each period, \( t , \) a generation of \( L_t \) individuals joins the labor force. Each individual has a single parent. Individuals within a generation are identical in their preferences and levels of human capital. Members of generation \( t \) live for three periods. In the first period (childhood), \( t - 1, \) individuals are endowed with single unit of time that is production. The set up of two sectors that produce the same output but employ different factors of production is in the spirit of Galor and Zeira (1993).
allocated by their parent between schooling and labor-force participation. Children can offer only $\theta \in (0, 1)$ units of raw labor due to their (purportedly) inferior physical ability and can work only in the traditional sector. Their earnings accrue to the parent. In the second period of life (parenthood), $t$, individuals save their income and allocate their single unit of time between childrearing and labor-force participation. They choose the number of children and the children’s time allocation between schooling and labor; they then direct their own remaining time to the labor market. They decide whether to supply raw labor (and to work in the traditional sector), or to supply efficiency units of labor (and to work in the modern sector). The decision is made according to the number of efficiency units of labor, $h_t$, they have. Specifically, they will choose the sector that maximizes their income, that is:

$$I_t = \max \{ \bar{w} h_t, w_r \},$$

where $I_t$ is potential income. Figure 1a shows the threshold in terms of $h_t$. In the third period, this generation consumes its savings with the accrued interest.

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8 “Children rarely receive an income even approaching the minimum wage, and their earnings are consistently lower than those of adults, even where the two groups are engaged in the same tasks” (Baquele and Boyden, 1988, pp. 4–5). Thus, $\theta$ can also be interpreted as discrimination against children in the labor market in the sense that they get less than their marginal productivity. For evidence see, for example, Baquele and Boyden (1988, Chapter 5).

9 The literature on child labor suggests that children are usually employed in industries where technologies are simple and production labor-intensive. Many studies show that the development of capital-intensive production has the effect of displacing child labor. See, e.g., Baquele and Boyden (1988). Galbi (1994) shows the same impact for the industrial revolution. Hence, we assume that children can be employed in the traditional sector only.

10 $I_t$ is potential income because it is the wage per one unit of time. However, parents devote some of their time to childrearing and hence earn only $(1 - z_n)I_t$. 
Figure 1. The Threshold Between the Modern and Traditional Sectors in Terms of Human Capital

1.a. Technology is constant

1.b. Exogenous technological progress
2.2.1. Preferences

We assume that individuals derive utility from consumption and from the potential income of their offspring in period $t + 1$. For the sake of simplicity, we assume that individuals consume only in the third period. Thus, the utility function of an individual who is a member of generation $t$ is: $^{11}$

$$u^t = c t+1 \ln(c_{t+1}) + (1 - \alpha) \ln(n_t I_{t+1}) \ ,$$

(7)

where $c_{t+1}$ is consumption in period $t + 1$, $n_t$ is the number of children of individual $t$ and $I_{t+1}$ is the potential income of each child in period $t + 1$, determined by the rule given in equation (6).

2.2.2. The budget constraint

As in Galor and Weil (1996, 1998), we follow the standard demand model of household fertility behavior. We assume that a parent faces a time constraint when choosing how many children to have. More specifically, we assume that time is the only input required in raising children. We denote by $z \in (0, 1)$ the amount of time needed to raise one child, implying that $(1/z)$ is the maximum number of children that can be raised.

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$^{11}$This form of preferences and utility function follows Galor and Weil (1998).
As mentioned earlier, it is the parent who allocates the time endowment of children between schooling and labor-force participation.\textsuperscript{12} Let $\tau_t \in [0, 1]$ be the fraction of time allocated to schooling and $(1 - \tau_t)$ the fraction of time allocated to labor-force participation of each child in period $t$. Thus, given the assumption on the physical ability of a child, each child supplies $\theta (1 - \tau_t)$ units of raw labor to labor-force participation.

Schooling is free\textsuperscript{13} and hence the only cost of schooling is the opportunity cost, i.e., the forgone earnings of the child. Therefore the budget constraint of the household is:

$$
(1 - zn_t) I_t + \theta (1 - \tau_t) n_t w^f = s_t .
$$

In the third period individuals consume their savings with accrued interest. Hence,

$$
c_{t+1} = s_t (1 + \bar{r}) .
$$

\subsection*{2.2.3. The production of human capital}

The level of human capital of member of generation $t + 1$, $h_{t+1}$, is predetermined in period $t$ through schooling. We assume that an individual is born with some basic human capital and can achieve more by attending school. As in Galor and Weil (1998) we assume that the level of human capital is an increasing, strictly concave function of the time devoted to schooling. In order to simplify, we assume that the production function of human capital is:

\textsuperscript{12} Parents do not discriminate between children: each child gets the same schooling as its siblings.

\textsuperscript{13} Introducing direct schooling costs does not change the qualitative result of the model, as long as they are constant. Kanbargi (1988) shows that in some Indian states, where education
\[ h_{t+1} = h(\tau) = a(b + \tau)\beta, \]

where \( a, b > 0 \) are constants and \( \beta \in (0, 1) \) is the “adjusted” elasticity of human capital with respect to schooling.\(^{14}\) Note that since \( \tau \in [0, 1] \), the level of human capital is bounded from below by \( ab^\beta \), the level of human capital that the child is born with, and from above by \( a(b + 1)^\beta \), the maximum level of human capital that can be achieved if the child’s time is allocated entirely to schooling.

2.2.4. Optimization

A member of generation \( t \) chooses the number of her children, the time allocation of her children between schooling and labor-force participation, and consumption, so as to maximize her utility function (7) subject to her budget constraint (8) and the constraints on \( n_t \) and \( \tau_t \), that is, \( n_t \in [0, 1/z]\(^{15}\) and \( \tau_t \in [0, 1] \). Substituting (8) and (9) into (7), the optimization problem facing the individual of generation \( t \) is:

\[
(n_t, \tau_t) = \arg\max \{ \alpha \ln \{(1 + \bar{F})[(1 - zn_t)I_t + \beta(1 - \tau_t)n_tw^e] + (1 - \alpha)\ln(n_tI_{t+1})\} \]
\]

s.t.:
\[
0 \leq \tau_t \leq 1
\]
\[
0 \leq n_t \leq 1/z.
\]

---

\(^{14}\) By “adjusted” we mean that \( \beta \) includes not only schooling but also innate ability, \( b \). Note that if we omit the parameter \( b, \beta \) will be exactly the elasticity of human capital with respect to schooling.

\(^{15}\) We ignore integer problems and allow the number of children per household to be in the segment \([0, 1/z]\).
Let us now describe the solution to this problem. Note that $h_t$ is determined in period $t - 1$ and hence the parent chooses her labor factor supply independently of the optimization problem, according to equation (6), that is, $I_t$ is exogenous to the maximization problem. There can be two different cases: (i) The parent supplies raw labor and earns $(1 - zn_t)w^c$, and (ii) the parent supplies efficiency units of labor and earns $(1 - zn_t)\bar{w} h_t$. Each case is solved separately. In each case the parent considers the possibility that her children will work in the modern sector in the next period, i.e., she assumes $I_{t+1} = \bar{w} h_{t+1}$. She maximizes (11) with respect to $n_t$, $\tau_t$. She substitutes $\tau_t$ into the production function of human capital [equation (10)] and calculates $h_{t+1}$. Then she substitutes $h_{t+1}$ into (6) and finds $I_{t+1}$. If $\max \{ \bar{w} h_{t+1}, w^c \} = \bar{w} h_{t+1}$, then $(n_t, \tau_t)$ are the solutions to the problem. If $\max \{ \bar{w} h_{t+1}, w^c \} = w^c$, then $\tau_t = 0$ and the parent maximizes (11) with respect to $n_t$ only. Note that if $\max \{ \bar{w} h_{t+1}, w^c \} = w^c$ then $\tau_t = 0$ is optimal, because any fraction of time devoted to schooling is chosen only to maximize children’s future potential income. If future potential income is $w^c$, then education is a waste of time.$^{16}$

**Assumption 1**: $\alpha \varepsilon > \theta$.

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$^{16}$ Unpalatable as it may seem, this is probably true. See Baquele and Boyden (1988), especially p. 6; Bonnet (1993); and Grootaert and Kanbur (1995), p. 193. One might wonder why parents do not send their children to school and benefit from transfers from them later in life (a Pareto-improving mechanism under the basic model). For simplicity we counterfactually assume that such a mechanism is not credible. Alternatively, we could introduce subsistence-level consumption in period 2 and restrict borrowing for consumption, but doing so will only encumber the model without adding to its predictive power. When we introduce endogenous technological progress (Section 3.2) that depends on the preceding period’s level of human capital, the mechanism is no longer optimal and hence no counterfactual assumption is needed.
This assumption is needed to ensure that \((1 - \alpha)/(z - \theta) < 1/z\), i.e., that the parent devotes some time to labor-force participation.\(^{17}\)

Define \(q = \left( \frac{w^c}{\frac{aw}{v}} \right)^{\frac{1}{\beta}} \left( \frac{1}{\beta} - 1/b + 1 + b \right) \).

**Proposition 1:** Under assumption 1, if \(\theta q > z\), then the solution to the maximization problem is:

\[
\tau_t = \begin{cases} 
0 & \text{if } h_t < \tilde{h} \\
\frac{\beta z w h_t - \beta w^c \theta - bw^c \theta}{w^c \theta (1 - \beta)} & \text{if } \tilde{h} \leq h_t \leq \frac{w^c \theta (1 + b)}{\beta z w} \\
1 & \text{if } h_t \geq \frac{w^c \theta (1 + b)}{\beta z w}
\end{cases}
\]  

(12)

and

\[
n_t = \begin{cases} 
\frac{1 - \alpha}{z - \theta} & \text{if } h_t \leq \frac{w^c}{w} \\
\frac{(1 - \alpha)wh_t}{z \bar{h} - w^c \theta} & \text{if } \frac{w^c}{w} \leq h_t \leq \tilde{h} \\
\frac{(1 - \alpha)(1 - \beta)\bar{h}^2 w h_t}{z \bar{h} - w^c \theta (1 + b)} & \text{if } \tilde{h} \leq h_t \leq \frac{w^c \theta (1 + b)}{\beta z w} \\
\frac{1 - \alpha}{z} & \text{if } h_t \geq \frac{w^c \theta (1 + b)}{\beta z w}
\end{cases}
\]

\(^{17}\) A less restrictive assumption, crucial to the solution of the maximization problem, is \(z > \theta\). This means that a child costs more than it can earn. Otherwise, the problem is not well defined. Cunningham (1990, p. 125) claims that this was probably the case in 18th- and 19th-century England.
where \( \bar{h} \equiv \frac{q \theta v^e}{z w} \).

Otherwise,

\[
\tau_i = \begin{cases} 
\frac{\beta v - \beta \theta - b \theta}{\theta (1 - \beta)} & \text{if } h_i \leq \frac{w^e}{w} \\
\frac{\beta \bar{w} h_i - \beta w^e \theta - b w^e \theta}{w^e \theta (1 - \beta)} & \text{if } \frac{w^e}{w} \leq h_i \leq \frac{w^e \theta (1 + b)}{\beta \bar{w}} \\
1 & \text{if } h_i \geq \frac{w^e \theta (1 + b)}{\beta \bar{w}}
\end{cases}
\] (13)

and

\[
n_i = \begin{cases} 
\frac{(1 - \alpha)(1 - \beta)}{z - \theta (1 + b)} & \text{if } h_i \leq \frac{w^e}{w} \\
\frac{(1 - \alpha)(1 - \beta)\bar{w} h_i}{\bar{w} h_i - w^e \theta (1 + b)} & \text{if } \frac{w^e}{w} \leq h_i \leq \frac{w^e \theta (1 + b)}{\beta \bar{w}} \\
\frac{1 - \alpha}{z} & \text{if } h_i \geq \frac{w^e \theta (1 + b)}{\beta \bar{w}}
\end{cases}
\]

The **Proof** follows from the Kuhn-Tucker theorem.

**PROPOSITION 2:** Schooling is a non-decreasing function of the parent’s income, and fertility is a non-increasing function of the parent’s income, i.e., \( \partial \tau_i / \partial h_i \geq 0 \) and \( \partial n_i / \partial h_i \leq 0 \).

The **Proof** follows from the differentiation of (12) and (13) with respect to \( h_i \).
**Assumption 2:** \( 1 - \alpha > z - \theta \).

This assumption is needed to assure that at the highest rate of fertility the population does not contract.

It follows from the solution to the household’s maximization problem that as long as child labor exists, the optimum number of children is greater than \( (1 - \alpha)/z \), which is the optimum number of children without child labor.\(^{18}\) Hence, child labor increases fertility. Moreover, as the parent’s income increases, thereby increasing her relative income (to the child’s income), the optimum number of children declines. Along with the decline in the number of children, the time allocated to children’s schooling increases because the relative importance of children’s earnings declines. This result might explain a familiar feature of demographic transition: a rapid decline in fertility accompanied by higher rates of growth in output per capita. It also implies a trade-off between quantity and quality of children and that as the economy develops, individuals prefer quality to quantity.

### 2.3. The Dynamic System

The level of human capital in period \( t + 1 \), \( h_{t+1} \), is uniquely determined by the time allocated to schooling in period \( t \). Since \( \tau_t \) is uniquely determined by the level of human capital in period \( t \), \( h_t \), the level of human capital in period \( t + 1 \), \( h_{t+1} \), is a real valued function of \( h_t \). Thus, the solution of the maximization problem in each period generates a first-order, nonlinear dynamic system in \( h_t \), denoted here by \( \Psi(h_t) \), which is given by substituting \( \tau_t \) from (12) or (13) into (10).

\(^{18}\) Note that if the maximization problem was formulated without child labor, i.e., with the same utility function, but a different budget constraint, \( (1 - zn_i)I_t = c_t \), the optimum number of children would be \( (1 - \alpha)/z \), regardless of \( I_t \).
**Proposition 3:** If $\theta q < z$, i.e., if $\tau_i$ is given by (13), then the dynamic system, $\mathcal{H}(h_i)$, is strictly concave, strictly monotonically increasing and there exists a unique steady state equilibrium.

**Proof:** First, note that $\mathcal{H}(0) > 0$. Secondly, since $\mathcal{H}'(h_i) > 0$ and $\mathcal{H}''(h_i) < 0$, $\mathcal{H}(h_i)$ is strictly concave. Finally, $\mathcal{H}(h_i)$ is bounded from above. Thus, there exist a unique $\bar{h}$ such that $\mathcal{H}(\bar{h}) = \bar{h}$.

**Proposition 4:** If $\theta q > z$, i.e., if $\tau_i$ is given by (12), then there can be either multiple equilibria or a unique equilibrium.

**Proof:** Note that for $h_i \in [0, \tilde{h})$, $\mathcal{H}(h_i)$ is constant and equal to $ab^\beta$. Hence, the existence of a low equilibrium depends on whether $\tilde{h} > ab^\beta$ or not. $\tilde{h}$ is a point of discontinuity because $\tau_i$ changes from 0 to a positive value and $\lim_{h_i \to \tilde{h}^+} \mathcal{H}(h_i) > ab^\beta$. If $\mathcal{H}(\tilde{h}) > \tilde{h}$, there is a high equilibrium because $\mathcal{H}(h_i)$ is bounded from above. If not, $\mathcal{H}(h_i)$ may or may not intersect the 45° line. If it does intersect the 45° line, it must do so it twice; the first intersection will be an unstable equilibrium and the second will be a stable equilibrium. It is clear, though, that at least one equilibrium exists, because if $\tilde{h} > ab^\beta$, the low equilibrium exists. Otherwise, a high equilibrium exists because $\mathcal{H}(h_i)$ is bounded from above.

The five possible shapes of $\mathcal{H}(h_i)$ are drawn in Figure 2 and can be divided into three groups: (i) The dynamic systems drawn in Figures 2a and 2b. Both these systems have unique and stable equilibria characterized by high income, a small number of
children in each household, and almost no child labor.\textsuperscript{19} (ii) Figure 2c, where equilibrium is also unique and stable, but is characterized by low income, a large number of children in each household, and extensive child labor.\textsuperscript{20} Note that for these two groups, the initial condition of the economy, i.e., the level of human capital at time 0, $h_0$, has no effect on the long-run equilibrium. (iii) The third group consists of the dynamic systems drawn in Figures 2d and 2e. In Figure 2d, there are three steady state equilibria: the low and the high ones are stable, and the “middle” one is unstable; in Figure 2e only the low and high steady-state equilibria exist. For these two dynamic systems, the initial level of human capital is crucial because it determines the characteristics of the long-run equilibrium.

At this point, we will not extend the discussion to the parameters of the model that characterize each of these five possible cases. As we will see in Section 3, technological progress will ensure the convergence of the economy to the high equilibrium regardless of the initial condition for all the different possible cases.

\textsuperscript{19} The parameters can be adjusted so that child labor is abolished in the high equilibrium case.

\textsuperscript{20} Actually in this steady state children work all the time and get no education (see Grootaert and Kanbur, 1995, p.191).
Figure 2. Possible Shapes of $\Psi(h_t)$ When $\theta q > z$

2.a The shape of $\Psi(h_t)$ when $\theta q < z$

2.b.

2.c.

2.d.

2.e.
3. Technological Progress

3.1. Exogenous Technological Progress

Suppose the dynamic system that characterizes the evolution of the economy is given by Figure 2c. In this case, if technology is stationary the economy is trapped in poverty: low output, high fertility and a low level of human capital (which means that children spend all their time working and no time studying). In this section, we show that technological change will eventually eliminate the development trap and that the economy will converge to a high steady state.

Suppose that in each period the economy experiences an exogenous technological progress that augments labor efficiency (hence, this change occurs only in the modern sector). This means that equation (2) becomes:

\[ Y_{2,t} = F[K_t, (1 + \lambda)^t H_t] \tag{14} \]

where \( \lambda \) is the rate of technological progress in every period. It follows from (14) that equation (6) becomes:

\[ I_t = \max\{(1 + \lambda)^t \bar{w} h_t, w^c\} \tag{15} \]

Note that since \( w^c \) is constant and \( (1 + \lambda)^t \bar{w} h_t \) is growing over time, there exists a \( t \) such that \( (1 + \lambda)^t \bar{w} h_t > w^c \) (see Figure 1b). Denote this \( t \) by \( \tilde{t} \). Suppose the economy is at period \( \tilde{t} - 1 \). An individual who supplies raw labor and works in the traditional sector solves the maximization problem [the first line of (13) gives the solution]. She finds it optimal to provide her children with some schooling. And since incomes in period \( \tilde{t} \) will be no less than they were in period \( \tilde{t} - 1 \), at time \( \tilde{t} \) parents will also be able to provide their children with schooling for at least as long as in period \( \tilde{t} - 1 \). Note that since \( \tau_t \) is a function of \( I_t \), \( \tau_t \) is now a function of \( t \) and hence the dynamic system depends on \( t \), i.e., the dynamic system is nonautonomous.
Graphically, this means that the concave segment of $\Psi(h_t)$ shifts to the northwest in each period and that the horizontal segment shortens in each period. Note also that due to technological progress, in every period a smaller $\tau_i$ satisfies the threshold condition given by (15) and hence the discontinuity of $\Psi(h_t)$ shrinks in each period until it vanishes. Figure 3 describes the dynamic system under the exogenous technological progress.

**Figure 3. The Dynamic System Under Exogenous Technological Progress**

![Diagram showing the dynamics of $h_{t+1}$ and $\Psi(h_t)$]

a The dynamic system is non-autonomous. The concave segment moves to the northwest in every period.

At this point it is only natural to explore the evolution of consumption, fertility and schooling along the path(s) to the steady state. These paths are drawn in Figure 4: the economy is stationary until period $\tilde{t} - 1$. 
Figure 4. Consumption, Fertility and Schooling Along the Path to Steady State Under Exogenous Technological Progress

Consumption, fertility and schooling are constant. At period $\bar{t} - 1$, spending some positive time in schooling is optimal due to higher income in the next period. Therefore, the number of children falls and children are sent to school part-time. Note that consumption does not change in this period since household income remains unchanged at $\alpha m^f$ (although the number of children falls and they do not work full-time, having fewer children means that a parent can supply more labor in the market). From this period on, consumption and schooling grow every period while fertility declines. The mechanism behind these paths was explained in the Introduction. The increase in consumption is obvious: income increases and consumption is a fraction of income ($\alpha$ times income).
3.2. **Endogenous Technological Progress**

A more realistic assumption is that technology does not grow at a fixed rate, but rather reflects changes in other economic and demographic variables. In this section, we assume a specific function of technology, whose formulation is based on an observable phenomenon: technology progressed more in areas where population was denser.\(^{21}\) Furthermore, we assume that technology is significantly affected by the population’s human capital: the higher the level of human capital, the higher the level of technology. For simplicity, we assume that technology is perfectly passed on from one generation to the next, and thus the level of technology is non-decreasing over time. Formally, we assume that \(\lambda_{t+1}\), the level of technology at period \(t + 1\), is given by:

\[
\lambda_{t+1} = \max\{\lambda_t, L_h_t\}. \tag{16}
\]

Under this specification of the level of technology, we observe four stages of development. The first is characterized by locally stable equilibrium: Consumption, fertility, schooling and technology are stationary at their “traditional” levels. The second is characterized by stationarity of all the real variables as well as fertility and schooling at the same level as in the first stage, accompanied by technological progress that has no impact on the economy at this stage.\(^{22}\) In the third stage of development, the economy enjoys the fruits of the technological progress that took place in the previous stage, Namely, the level of technology has reached the point where the modern sector is launched. Both consumption and schooling increase, while fertility declines. In the fourth stage, two different paths are possible: If the minimum level of fertility exceeds one, child labor is abolished and technology, output and consumption grow forever. But

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\(^{21}\) Kremer (1993) argues that regions that started with larger initial populations experienced faster technological progress.

\(^{22}\) Footnote 27 discusses this result and provides a way to interpret it.
if the minimum level of fertility is less than one, technology, output and consumption reach a stationary level.\(^\text{23}\) In this case, child labor is abolished and fertility reaches its minimum level only if \( \bar{\lambda} \) (\( \bar{\lambda} = \sup \{\lambda_t\}_{t=0}^\infty \)) is sufficiently high.\(^\text{24}\)

The sequence \( \{\lambda_t\}_{t=0}^\infty \) is uniquely determined by \( \{(n_t, \tau_t)\}_{t=0}^\infty \) and by the initial conditions of the economy, \( (h_0, L_0) \). The size of the population in period \( t + 1 \), \( L_{t+1} \), is the product of the size of the population in period \( t \) and the number of children in each household in that period, that is:

\[
L_{t+1} = L_t n_t.
\]

(17)

Suppose that at \( t = 0 \) the economy is at its incipient stage of development, that is, \( h_0 = ab^\beta \text{,} \)\(^\text{25}\) and \( \lambda_0 > L_0ab^\beta. \)\(^\text{26}\) Note that as long as \( n_t > 1 \), the population grows over time. It follows from Assumption 2 that at the incipient stage of development the population increases in every period and therefore there exists some period \( t \) such that \( L_tab^\beta > \lambda_0. \) Until then, technology is constant and hence consumption, fertility and schooling are constant as well. This is the first stage of development mentioned above.

From this period on, technology increases due to population growth, but consumption, fertility and schooling remain constant at their initial level. It is important to emphasize that technological progress affects only the (as yet latent) modern sector.\(^\text{27}\)

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\(^{23}\) The results regarding the fourth stage of development depend crucially on the specification of \( \lambda_c \).

\(^{24}\) As noted in Section 2, \( (n_t, \tau_t) \) are determined by the parent’s income. In this case, \( \bar{\lambda} \) determines the parent’s income.

\(^{25}\) We can also assume that \( h_0 = 0 \) since \( h_1 = ab^\beta \) [as per equation (10)].

\(^{26}\) Note that we should assume that \( \lambda_0 \bar{w} h_r < w^c \) for all \( h_r. \) Hence the mechanism discussed in footnote 16 is no longer a Pareto improvement.

\(^{27}\) If there were a modern sector in the economy, wages would increase in that sector, but at this stage of development the existence of a modern sector is not optimal.
These characteristics prevail until the technology level is such that the threshold condition is satisfied, that is, \( \lambda_i \overline{w} h_i > w^e \). Again, as in Section 3.1, we denote this \( t \) by \( \tilde{t} \). It is easy to see that until the threshold condition is satisfied, \( n_t \) is constant and equal to \( (1 - \alpha)/(z - \theta) \); therefore, \( \tilde{t} \) is the smallest \( t \) that satisfies the following inequality:

\[
L_0 \left( \frac{1-\alpha}{z-\theta} \right) (ab^\theta)^{\frac{1}{w}} > w^e. \tag{18}
\]

By applying the same rationale as in Section 3.1, at period \( \tilde{t} - 1 \) the individual finds it optimal to reduce the number of children and to allocate some of their time to schooling. Hence, stage two refers to all the periods between the end of stage one and period \( \tilde{t} - 1 \).

From period \( \tilde{t} \) on, the level of technology increases due to the increase in the level of human capital. If \( n_t > 1 \), technology increases not only because of the level of human capital but also because of the increase in the size of the population.\(^{28}\) Thus, the third stage we identified begins from period \( \tilde{t} - 1 \): schooling increases and fertility declines, whereas consumption increases.\(^{29}\)

\[\text{At this stage of development it makes sense to assume that } n_t > 1 \text{ because the economy is only at the beginning of experiencing economic growth. The empirical evidence suggests that at this stage of development populations do increase. Furthermore, it should not be inferred from the specification of } \lambda_t \text{ that an increase in the number of children will increase the level of technology from this stage of development on, but rather that there exists a scale effect, i.e., if the number of households increases, technology will advance.}\]

\[\text{Note that consumption increases only from period } \tilde{t}. \text{ This point is explained extensively in Section 3.1. The shift from exogenous to endogenous technological progress has no effect at this point.}\]
As mentioned, \( h(t) \) is bounded from above and thus technological progress will continue forever only if the population increases over time, that is, if \( 1 - \alpha > z \). Otherwise, the level of technology is bounded from above by some \( \overline{\lambda} \).\(^\text{30}\) Note also that as long as \( n_t > 1 \), \( \lambda_t = \left( L_0 \prod_{s=0}^{t} n_s \right) h_t \).

The fourth stage of development depends crucially on the size of the population. If fertility, even at its lowest rate, assures that the population is non-decreasing over time, child labor is abolished and fertility reaches its lowest rate, that is, \( n_t \) and \( \tau_t \) reach a steady state, while technology, output and consumption grow forever. However, if at any stage the population starts declining; then technology output and consumption will reach steady state as well.\(^\text{31}\) The four stages of development are illustrated in Figure 5.

\(^{30}\) This follows immediately from the formulation of technology and the fact that there exists some period \( \bar{t} \) such that for all \( t > \bar{t} \) the population is decreasing and human capital is constant.

\(^{31}\) If we consider the case of working college students as an example of “child” labor, then (in the terminology of the model) \( \tau_t < 1 \), and the prediction that the level of technology might reach steady state can not be tested.
4. Concluding Remarks

This paper explores the relations between child labor, fertility and growth. It presents a unified model in which child labor and fertility are endogenously determined and explain the impact of these variables on economic growth. The model generates an endogenous demographic transition that occurs due to technological progress. This transition is accompanied by accumulation of human capital and a decline in child labor that is consistent with the process of development in developed countries for the past one hundred and fifty years.
The model abstracts from many factors related to child labor, fertility and growth. One essential factor is compulsory education. By the end of the 19th century education was compulsory in virtually all developed countries. Nowadays, education is compulsory in many less developed countries. Child labor legislation is another tool used to combat abuses of child labor. Nevertheless, economic incentives still rule the process of development. Inferior education systems and lack of better employment prospects usually push parents to opt for sending their children to work rather than to school. Thus, improving the educational system seems to be more efficient in combating child labor than compulsory education.

Finally, we argue that our model can also explain the extremely high fertility rate of ultra-orthodox Jews in Israel. Data from Berman (1998) show that the total fertility rate (TFR) in 1995/6 of this community was 7.61, which is even higher than the average TFR in Africa (6.0, according to CIHI, 1996), the continent with the highest TFR in 1995/6. Family income in the ultra-orthodox sub-population is very low compared to the entire population in Israel. A major source of income among the ultra-orthodox is child allowances received from the State, which account for 19.7% of the monthly household income in families where the prime-age male works, and for 31.8% of the monthly household income in families where the prime-age male attends yeshiva and does not work. These figures should be compared with the 7.4% that child allowances

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32 Berman (1998, p. 11) argues that “the proportion of ultra-orthodox Jewish males (aged 25–54) not working because of full time yeshiva attendance is high and rising: from 41% in 1980 to 60% by 1996.” His data also show that the income of households whose prime-age male attends yeshiva and does not work is 42% of the average monthly household income in Israel. The income of households within this community whose prime-age male works is 75% of the average monthly household income in Israel.

33 Hence, in this context, child allowances are analogous to child labor. In Israel, child allowances granted by the National Insurance Institute are a convex function of the number of children per household and thus encourage higher fertility rates.
contribute to the monthly household income in the entire population. Thus, the relatively high contribution of children to household income among the ultra-orthodox may explain their outstanding fertility rates.\textsuperscript{34}

\footnotesize
\textsuperscript{34} Of course, other factors affect fertility in this community, not least, perhaps, being the Commandment “be fruitful and multiply”. 
**Bibliography**


